A PHOSPHORUS RESIDENCE TIME MODEL: THEORY AND APPLICATION

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Chemical residence time has been used as a basis for modeling the chemical content of the ocean for some time (Barth, 1952). Recently, Vollenweider (1969) and other workers have used what is the chemical residence time approach to model the rate of recovery of several lakes following pollution abatement. The purpose of this paper is to relate the theoretical basis for the chemical residence time model, as it applies to phosphorus, discuss the limits, capabilities and applications of the model.

DEVELOPMENT OF THE MODEL

In some early attempts to model chemical concentrations in lakes it was assumed that from the standpoint of long term trends, a take may be likened to a completely mixed reactor subjected to continual and constant chemical influx. Losses from the reactor occur only through the outlet. Continuity considerations then give

$$V\frac{dc}{dt} = Qc_i - Qc \qquad (1)$$

where V is the volume of the lake, L^3; Q is the volumetric flow rate, L^3T^{-1}; c_i is the constant influx concentration, $M\,L^{-3}$ and c is the concentration in the lake, $M\,L^{-3}$. Integrating and applying the boundary condition that at time $= 0$, $c = c_0$ yields

$$c = c_i + (c_0 - c_i)e^{-t/R_w} \qquad (2)$$

where $R_w = VQ^{-1}$, the hydraulic residence time.

Equation (2) shows that following a decrease (or increase) in the chemical influx, steady-state conditions are approached exponentially in accordance with the hydraulic residence time of the basin and that three hydraulic residence times would be required for a lake to reach 95% of its new steady-state concentration. Thus, based on considerations of this type, it has been estimated that 90 yr would be required for Lake Michigan to achieve a 95% response to a decrease in phosphorus input (Rainey, 1967).

Although the hydraulic residence time model may be applicable for conservative elements, it has serious deficiencies when nonconservative elements, such as phosphorus, are considered. The model has two characteristics which should be noted; the steady-state concentration in the lake is identical to the input concentration, and the losses from the lake occur only through the outlet. Neither of these conditions apply to many lakes. Mean annual concentrations in lakes are often much lower than mean annual input concentrations, and the major loss of phosphorus frequently results from deposition to the sediments, not from discharge through the outlet.

For non-conservative elements, the validity of the model is improved if internal losses as well as outwash losses are taken into account in the mass balance. Thus, equation (1) must be modified to account for the reactivity of the element in the lake. Studies have shown that the sediments of lakes act as sinks or traps for phosphorus. Although there is some release of phosphorus from sediments, the net flux of phosphorus over an annual cycle is to the sediments. This follows from the fact that a significant portion of the organisms that settle onto the bottom are refractory so that regeneration of the phosphorus upon mineralization is less than 100 percent (Jewell, 1971).

To account for internal losses, equation (1) is modified to

$$V\frac{dc}{dt} = Qc_i - Qc - kcV \qquad (3)$$

when k is the internal loss rate constant, T^{-1}.

The rate of internal loss is written as a first order reaction, in which the loss is directly proportional to the mean content of the lake. This assumption is reasonable, at least as a first approximation (see Vollenweider, 1969). In the case of phosphorus, k accounts for the net loss of P to the sediments and kCV is the net phosphorus sedimentation rate. The above modification could also be adapted to nutrients such as nitrogen and carbon, but the relationship would necessarily be more complex since a gas phase must be considered in the aqueous chemistry of nitrogen and carbon. Equation (3) may be potentially useful as a model for the silica cycle in natural waters, however.

In a stratified lake the possibility that the surface layer may contain different amounts of a non-conservative chemical than the bottom waters due to biological, chemical or physical processes must also be considered. For example, phosphorus concentrations in the surface water of a stratified lake during certain

times of the year may be only a fraction of the average concentration over the whole lake on account of its removal from the surface layer by biological and chemical processes. Thus the outwash concentration during those periods of the year when the lake is not well mixed may be significantly different than the average concentration over the whole lake. In many situations this effect will be of minor significance when an annual cycle is considered, particularly when the hydraulic residence time is long in relation to the period of stratification. However, in those situations when stratification must be taken into account, Equation (3) may be modified in the following manner:

$$V \frac{dc}{dt} = Qc_i - Q\alpha c - kVc \qquad (4)$$

where α = dimensionless proportionality factor that relates the mean annual outwash or surface water concentration to the mean annual concentration over the whole lake. Consequently it is assumed that the average annual outwash concentration is directly related to the annual concentration over the whole lake by a constant factor. Since the total phosphorus concentration in the epilimnion of most lakes is less than the hypolimnion concentration, α will generally be less than one. For example over the last two years the average concentration of total phosphorus in the surface waters of Lake Mendota, Madison, Wisconsin, was about 70% of the average concentration for the entire lake. Thus α would have a value of about 0.7 for Lake Mendota. For lakes which do not stratify, α would be equal to unity. Note that in the above models, it has been assumed that the water balance is such that yearly inflow equals yearly outflow. Dingman and Johnson (1971) have more properly equated the water balance equation with the mass balance equation, but for the purposes of this paper the above simplification will be utilized.

RESPONSE T0 CHANGES IN NUTRIENT INFLUX

Of special interest to those concerned with lake management is the rate of improvement which might be expected as a result of reduced nutrient loadings (or vice versa) to a lake. For example, how fast and to what extent will a lake respond to a step-change in the phosphorus influx, such as would result from improved wastewater treatment? Equation 4 may be applied directly to gain insight regarding the anticipated response. Rearranging Equation (4) gives

$$dc + \frac{(Q\alpha + kV)}{V} c \, dt = \frac{Q}{V} c_i \, dt \qquad (5)$$

By substitution, this can be simplified to:

$$dc + \frac{1}{R_p} c \, dt = \frac{1}{R_w} c_i \, dt \qquad (6)$$

where

$$R_p = \frac{V}{Q\alpha + kV},$$

which is the phosphorus residence time for the lake. Equation (6) may be integrated directly, and if at

$t = 0, c = c_0,$ then

$$c = c_i \frac{R_p}{R_w} - \left(c_i \frac{R_p}{R_w} - c_0 \right) e^{-t/R_p}. \qquad (7)$$

The steady-state concentration in the lake, c_∞, for this model is not identical to the input concentration, but differs by a factor which is the ratio of the phosphorus and hydraulic residence times.

$$c_\infty = c_i \frac{R_p}{R_w} \qquad (8)$$

Equation (7) may then be expressed in terms of the ultimate steady-state concentration by

$$c = c_\infty - (c_\infty - c_0)e^{-t/R_p} \qquad (9)$$

or

$$\frac{(c - c_\infty)}{(c_0 - c_\infty)} = e^{-t/R_p} \qquad (9a)$$

Equations (8) and (9) can be used to estimate the gross response of a lake to a step reduction in the phosphorus influx. If the lake was at steady-state prior to the input reduction, it can be seen from Equation (8) that, ultimately, the mean content of the lake will be reduced in direct proportion to the change in the influx, i.e. a 50% reduction in the input will result in a 50% reduction in the mean content of the lake. Equation (9) shows that the new steady-state concentration will be approached exponentially as a function of the phosphorus residence time (Fig. 1). The time required to reach 50% of the expected change is $0.69 \, R_p$: 95% of the expected change will be reached in a period equal to three phosphorus residence times.
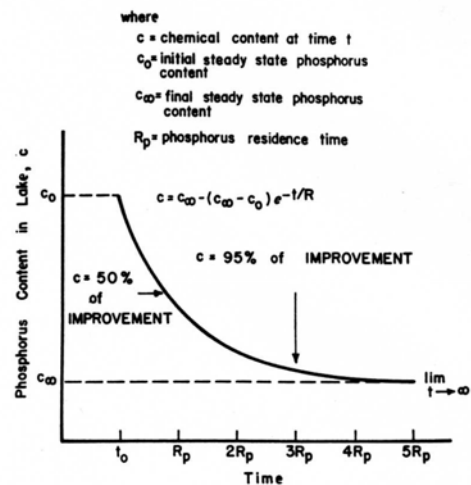

Fig. 1. Rate of recovery of a lake following a step-change reduction of the phosphorus influx.

DETERMINATION OF R_p

For steady-state conditions, R_p can be determined most conveniently from equation (8).

$$R_p = \frac{R_w c_\infty}{c_i} = \frac{V c_\infty}{Q c_i} \qquad (10)$$

Equation (10) shows that the phosphorus residence time for a lake which is in equilibrium with its input can be determined if the mean annual content, $V c_\infty$ is divided by the annual input to the lake, $Q c_i$. If it is assumed that the lake is at steady-state prior to the input reduction, then equation (10) may be used to compute R_p using data obtained before the input reduction. Since R_p is a function only of V, Q, k and α, R_p is not influenced by changes in the input concentration.

Once the phosphorus residence time is known, the rate of response to phosphorus abatement (or pollution) can be predicted from the model. The phosphorus residence time accounts for the overall sedimentation process as well as effects of stratification.

The use of chemical residence time circumvents determining the values of k and α. Alternatively, if k and α are known or estimated (as for example from phosphorus sedimentation rates) and V and Q are known, R_p can be calculated directly. If R_p, is known and the steady-state phosphorus content is known, then the input of phosphorus could be estimated by solving equation (10). This would be a possible method of roughly estimating the loading of phosphorus to a lake. It is interesting to note the relationship between the phosphorus residence time and the hydraulic residence time. From their basic definitions, it can be shown that

$$R_p = \frac{1}{\dfrac{\alpha}{R_w} + k} \qquad (11)$$

If the lake under consideration is well mixed ($\alpha = 1$), equation (11) shows that, for a given k, the larger the hydraulic residence time becomes the less effect it has on the phosphorus residence time. Thus, as R_w. becomes large, R_p approaches $1/k$. In contrast, as the water residence time becomes smaller, the chemical reactivity becomes less important and R_p approaches the value of R_w. Also, when k is small R_p approaches R_w, which indicates that for this condition phosphorus acts like a conservative substance. Plots of equation (11) for various values of k are given in Fig 2.

STEADY-STATE ASSUMPTIONS

A basic assumption of the model is that the influx of phosphorus to a lake is constant. Realistically, the annual influx of phosphorus to a lake may differ somewhat from year to year due to natural random variations in, for example, the annual amount of runoff
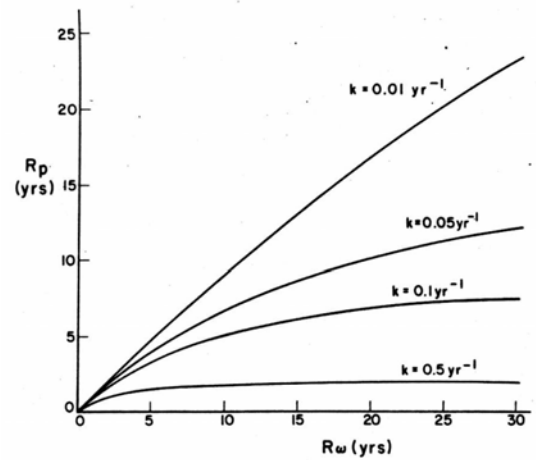

Fig. 2. Relationship between R_p and R_w $1/\{[(\alpha)/R_w] + k\}$ for hypothetical values of k ($\alpha = 1$).

or precipitation. Also, the influx may be changing continuously for some lakes, reflecting, for example, the steadily increasing use of phosphates by man. If the input of a chemical is continuously changing, the phosphorus concentration within the lake will not reach a steady-state concentration with the influx, but will lag continuously from the steady-state condition. If large, this lag could lead to significant errors in estimating R_p and the extent to which in-Lake concentration will change as a result of reduced influx.

To estimate the potential significance of continually increasing input concentrations, equation (6) was solved for the condition that the input concentration was a linear function of time rather than a constant. The effect of lag was then evaluated by comparing at a given time the concentration which would occur in the lake from a continuously increasing input to the steady-state concentration which would result if the input concentration was constant. For the conditions described, it can be shown that mean annual concentrations would lag the predicted steady-state value by an amount given by

$$lag = \Delta c_i \, \frac{R_p^2}{R_w} \qquad (12)$$

where Δc_i = the annual rate of increase in the input concentration, MT^{-1}. Since R_p is normally smaller than R_w often by a considerable amount, the error in concentration which results from steady-state assumptions is always smaller than the product of R_p and the yearly incremental increase in the input concentration. Since this annual increase is often small, long term temporal trends can be ignored in many practical situations.

CALCULATIONS OF R_p

Although the model was derived based on the mean annual phosphorus content (obtained from the average of systematic measurements over a whole year), the mean content at spring turnover or during the winter period might be used as the basis for the annual total

phosphorus content. During these periods, lakes are generally well mixed (except under ice cover) and biological productivity is often at an annual minimum. The use of concentrations obtained at the time of spring turnover to forecast whether a lake may be expected to produce excessive growths of algae or other aquatic plants is well known (Sawyer, 1947; Vollenweider, 1968). It is, therefore, reasonable that winter or spring-turnover phosphorus levels are proportional to the annual loading to the lake (see Vollenweider, 1968 for a discussion of this relationship). One obvious advantage of using winter or spring turnover measurements would be that one sample could be used to characterize the entire lake.

However, caution should be exercised when using spring turnover concentrations. It was observed at ice out (late April) of 1971 on Lake Mendota, that the total phosphorus content decreased by over 50% compared to the content measured just prior to ice out. The cause of this decrease was deduced to have been caused by a bloom of diatoms (the lake actually appeared brown to observers along the shore). The diatoms took up the available phosphorus (most of the total) and then settled to the bottom; the process removed a large portion of phosphorus from the water column. Phosphorus was subsequently released from the diatoms as they decomposed on the lake bottom. By June, when summer stratification first developed, the phosphorus content had increased to near the level measured before ice out and was again available for summer algae blooms. Although this may have been an unusual occurrence, such a possibility must be considered when measuring the spring turnover phosphorus level.

The phosphorus residence time has been calculated for a number of lakes where sufficient data is available. The results are given in Table 1 along with the hydraulic residence times of the respective lakes. Table 1 shows that in all cases the phosphorus residence time is less than the hydraulic residence time, indicating the lakes are acting, as a sink for phosphorus. It should be noted that although the chemical content of lakes, either the mean annual content or the mean vernal content, may be determined with reasonable reliability, the rate of phosphorus influx is often much less precisely known, especially when the influx is estimated from land use patterns and generalizations from the literature (see Sonzogni and Lee, 1974b). Nevertheless, the fact that all phosphorus residence times listed in Table 1 are less than the hydraulic residence times, even considering that they are crude estimates, gives strong evidence that the response of a lake to phosphorus abatement will be more rapid than that predicted from the hydraulic residence time.

TESTS OF THE MODEL

One of the best examples of a lake which has responded to a decreased nutrient flux is Lake Washington

Table 1. Estimated mean residence times for water and phosphorus for various lakes

Lake	Water Residence Time (R_w) years	Phosphorus Residence Time (R_p)	Source
Washington	3.2	0.8	Megard (1971)
Minnetonka	25	0.9	Megard (1971)
Sebasticook	3.5	1.4	Megard (1971)
Norrviken	0.6	0.3	Megard (1971)
Clear	6	2	Megard (1971)
Mendota	4.5	0.9	Sonzogni and Lee (1974)
Michigan	30	6	Lee (1972)
Erie	2.6	0.34	Dobson (1973)
Ontario	7.8	3.2	Dobson (1973)
Agerisee	8.7	2.3	Vollenweider (1969)*
Turlersee	2.2	0.7	Vollenweider (1959)
Hellwilersee	3.8	2.7	Vollenweider (1969)
Bodensee – Obersee	4.9	3.9	Vollenweider (1969)
Pfaflikersee	2.6	1.2	Vollenweider (1969)
Zurichsee – Untersee	1.5	1.2	Vollenweider (1969)
Greifensee	2.0	1.4	Vollenweider (1969)
Baldeggersee	4.5	3.1	Vollenweider (1969)

*R_p as calculated from Vollenweider (1969) based on mean vernal content instead of mean annual content.

ton (Edmondson, 1969; 1970). Between 1963 and 1968 sewage effluents were diverted from entering Lake Washington so that the annual input of phosphorus was reduced by about 50%. The response of the lake to the decreased nutrient influx was both prompt and sensitive. During the winter of 1969 the concentration of soluble orthophosphorus was only 28% of the winter concentration before diversion. Megard (1971) has compared the actual rate at which the phosphorus content decreased following diversion with that predicted from the phosphorus residence time model. The observed rate of decrease was parallel to the predicted rate and the 1969 measured concentration was similar to the predicted concentration for 1969. Megard found that the observed rate at which the phosphorus content decreased was parallel to the predicted rate, despite the fact that his model assumed that the complete diversion was done in one step whereas the diversion was actually completed in stages over a few years. Furthermore, he found that the measured mean annual concentration in 1969 was very close to the new steady-state concentration predicted by the model. In general, the Lake Washington case provides a reasonably successful test of the model.

The 1958 diversion of sewage effluent from Lake Waubesa and Lake Kegonsa, the third and fourth lakes, respectively, in the Yahara chain of lakes located near Madison, Wisconsin, provide other examples of the response of lakes to a decreased phosphorus loading. It is of interest to compare, using the limited data available, the observed rate at which the lakes responded to the diversion with that predicted based on the phosphorus residence time model (see Sonzogni and Lee, 1974a).

The hydraulic residence times of these shallow, well mixed takes were of the order of a few months so that,

as discussed previously, the phosphorus residence time may be assumed to be about the same as the hydraulic residence time.

For Lake Waubesa, the theoretical hydraulic residence time (and, consequently, R_p) was about 0.22 yr after diversion (Sonzogni and Lee, 1974). If sewage effluent is taken to have contributed 90% of soluble inorganic phosphorus (Sawyer et al., 1944) then according to the exponential recovery model, in less than a year after the December, 1958, sewage diversion the equilibrium phosphorus content should have been reduced by nearly 90%. Unfortunately, there is no information on what the soluble inorganic phosphorus was the winter immediately prior to the diversion. However, compared to 1950 and earlier the mean winter concentration 1 yr after diversion was reduced by over 75%.

Lake Kegonsa, which had a post diversion hydraulic residence time of about 0.31 yr, was thought to receive most of its soluble inorganic phosphorus input from Lake Waubesa (Sawyer el al. 1943; 1944), so that a 90% reduction in the phosphorus flux to Lake Waubesa should also have resulted in nearly a 90% reduction of the input to Lake Kegonsa. However, Kegonsa's rate of recovery depended not only on its own flushing period but also on that of Lake Waubesa and the reach of the Yahara River between the two lakes (this reach includes a small, shallow lake-like widening of the river). Since the combined theoretical hydraulic residence times is slightly greater than 0.51 yr, Lake Kegonsa should have reached 95% of its new equilibrium phosphorus concentration within two years. This presents an upper figure, since it has been assumed that Lake Kegonsa did not begin to recover until Lake Waubesa had equilibrated with the new input.

The first winter after diversion the data shows a small decrease in the soluble inorganic phosphorus concentration in the outlet of Lake Kegonsa compared to 1950 values, while the second winter after diversion the soluble inorganic phosphorus concentration was reduced by nearly 70%. Again, the lack of data immediately prior to diversion precludes a more accurate assessment of the situation, but it appears that the observed rate at which Lake Kegonsa equilibrated to the decreased chemical flux was close to the rate predicted by the exponential decay model. Thus, both Lake Waubesa and Lake Kegonsa appeared to have recovered at a rate corresponding to that predicted by the model.

Of considerable importance is the fact that the sediments of Lake Washington, Lake Kegonsa and Lake Waubesa did not act as major sources of phosphorus to the overlying waters following diversion. The Zellersee, a European lake, as cited by Vollenweider (1969), is another example of a lake whose sediments were not a source of phosphorus following a reduction of the phosphorus influx. As discussed by Lee (1970), shallow lakes such as Lake Waubesa and Lake Kegonsa which have received very large amounts of phosphorus should show the greatest overall release of phosphorus from the sediments due to the much higher wind and organism induced mixing between the sediments and the overlying waters. Consequently, the above examples present strong evidence against significant buffering effects of the sediments subsequent to a reduction of the phosphorus income of a lake.

PHOSPHORUS SEDIMENTATION RATE

It is of interest to compare the phosphorus deposition rate calculated from a mass balance [Equation (3)] with the rate estimated from sediment analyses. Taking. Lake Mendota as an example, the rate of phosphorus deposition during recent years was estimated from sediment core studies to be 1.19×10^3 mg Pm^{-2} yr^{-1} for the deep hole area (23 m) of the lake (Bortleson, 1971). This rate was found to increase toward shallower waters, the maximum rate (1.6 times the rate found for the deep hole area) observed at a twelve meter station. The average phosphorus sedimentation rate over the whole lake was not estimated by Bortleson (1971), but the rate might be expected to fall somewhere between the deep hole and 12 m rate.

At steady-state, equation (4) shows that the phosphorus sedimentation rate (kVc) is equal to the annual input rate Qc_i minus the annual outflow rate ($Q\alpha c$). Using an average flow rate of 80 mgd for Lake Mendota (MeCaskey, 1955), an α value of 0.7 (see previous discussion), and a mean annual phosphorus concentration of 0.12 mg L^{-1} P (Sonzogni and Lee, 1974a), the phosphorus sedimentation rate is calculated to be 1.35×10^3 mg Pm^{-1}yr^{-1}. This value is remarkably close to the rate estimated from Bortleson's (1971) sediment core studies.

Megard (1971) made similar calculations for Lake Washington and found excellent agreement between the phosphorus deposition rate as calculated from sediment analysis (1.5×10^3 mg Pin^{-2} yr^{-1}) and the rate estimated from a materials balance (1.7×10^3 mg Pm^{-2} yr^{-1}). While the close agreement of phosphorus sedimentation rates as calculated by the two independent methods may be fortuitous in view of the roughness of the data, the results support the basic mass balance relation and indicate that the model is not grossly in error.

UTILIZATION OF THE MODEL FOR LAKE MANAGEMENT

When considering the costly diversion or elimination of nutrient sources to a lake, it is often asked what effect such action will have on the water quality, especially with regard to the phosphorus level in the lake, and how fast this effect will take place. Lake Mendota, the first lake in the Yahara chain of lakes, provides an example of a eutrophic, dimictic lake whose phosphorus income was recently reduced by an estimated 20% (Sonzogni and Lee, 1974a). The effect of this reduction on the phosphorus content of the lake has not yet been

determined, although it is under investigation (Sonzogni, 1974). Because it is established that the sediments of Lake Mendota serve as a sink for phosphorus (Bortleson and Lee, 1972) the time needed to equilibrate to a new phosphorus flux should be more rapid than predicted from the hydraulic residence time (4.5 yr).

The phosphorus residence time for Lake Mendota may be estimated by dividing the mean annual steady-state phosphorus content by the annual phosphorus loading. The mean annual phosphorus content was determined for 1970-1971 (assuming the content at this time was an equilibrium content) from detailed sampling at nearly weekly intervals. The annual phosphorus loading for Lake Mendota was recently estimated by Sonzogni and Lee (1974b). It should be noted that the estimated annual phosphorus loading is only a rough approximation and may vary widely from year to year. Nonetheless, using this data, a mean phosphorus residence time of about 0.9 yr is obtained for Lake Mendota.

Since the change in Q due to the diversion was negligible, the mean phosphorus residence time for Lake Mendota should not have changed after diversion, so that a new steady-state phosphorus content should be reached about 3 years following the start of diversion. Thus, the total phosphorus content is predicted to decrease by about 20% about three years after diversion. The mean annual concentration of total phosphorus would decrease from about 0.12 to 0.10 mg L^{-1} P, or by about 0.02 mg L^{-1} P, as a result of the diversion.

Unfortunately, because of the relatively small decrease in the estimated phosphorus loading as a result of the 1971 diversion, it is doubtful that the predicted change in the mean annual phosphorus content will be seen analytically. Normal year to year variability in the loading from other sources, as well as other factors, may overshadow the effect of the diversion. Thus, the diversion will probably not serve as a significant test case for the model. This is not to say that the 1971 diversion was not of value. On the contrary, the diversion served essentially as a preventive measure to avoid future degradation of the water quality of the lake. This degradation could have occurred if waste water, steadily increasing in amount as a result of a rapidly expanding population on the north and west side of the lake, were to continue to be discharged into the lake.

Considerable attention has recently been focused on the eutrophication of Lake Michigan and the effect of curbing excessive phosphorus enrichment of the lake. Water pollution control agencies of the states bordering on Lake Michigan and the federal government (EPA) established in 1968 an 80% phosphorus removal standard for waste waters discharged into Lake Michigan. The criteria was essentially met as of December, 1972. This action has, according to an estimate recently made by Lee (1972), reduced the annual phosphorus influx to the lake by over 50%.

A question of great importance is the rate at which Lake Michigan will respond to this limitation of the phosphorus influx.

Because the hydraulic residence time of Lake Michigan is on the order of 30 yr, some persons have indicated that it may take nearly 100 yr or longer for Lake Michigan to equilibrate to the reduced phosphorus flux (Baumgartner, as cited by Risley, 1968). Such predictions are based for the most part on the hydraulic residence time model which necessarily assumes phosphorus behaves conservatively. However, in view of the long hydraulic residence time of Lake Michigan, it is felt that it is much more appropriate to use a residence time model based on the chemical reactivity of the element.

Recently, Lee (1972) has made an estimate of the phosphorus residence time for Lake Michigan, and thereby predicted the response of the lake to the reduced loading using the phosphorus residence time model. Using data from the Phosphorus Technical Committee to the Lake Michigan Enforcement Conference (Zar, 1972), a total loading of 18.1 million pounds yr^{-1} of phosphorus was assumed to enter the lake prior to the reduced loading. The steady-state content was calculated by assuming an average phosphorus concentration of 0.01 mg L^{-1}P (Sehelske, 1972) over the whole volume of the lake (5 x 10^{15} L; Hutchinson, 1957). Thus, it is computed that the phosphorus residence time in Lake Michigan is about 6 yr. Therefore, based on the phosphorus residence time model, it would be expected that about three residence times, or 15-20 yr, will be needed to achieve 95% of the expected change in the phosphorus content.

It should be noted that the above rate of recovery assumes that the phosphorus content of Lake Michigan in 1971 (before the decreased loading was accomplished) was in equilibrium with the phosphorus loading to the lake (Lee, 1972). The Phosphorus Technical Committee Report (Zar, 1972) presents data which show that, on the average, the total phosphorus concentration in the water supply intake of the Chicago Water Filtration Plant has been increasing at a significant rate during the past 15 yr, indicative of a small but steady rate of increase of the phosphorus loading. If this data is considered representative of the entire lake, it means that if the annual phosphorus load were to be maintained constant at the 1971 level, the phosphorus concentration would increase for several years before equilibrium is reached. Nevertheless, as was discussed earlier, even if the phosphorus concentrations were tending to increase as a result of a small, steady increase in loading prior to 1972, the assumption of a steady-state condition in 1971 is reasonable within the limits of the data. Moreover, the dramatic drop in the total phosphorus input (approximately 60%) during 1972 would completely overshadow the loading effect prior to 1972. This follows from the fact that the residence time model utilizes an exponential recovery with the greatest recovery immediately following the input reduction.

To be sure, a new steady-state concentration will be reached in much less time than predicted from a hydraulic residence time model.

It should be noted that the phosphorus concentration used in the above calculations may be high for Lake Michigan as a whole. Some of the data on the total phosphorus present in the open waters of the lake have a total phosphorus concentration of less than 0.01 mg L^{-1} P (Lee, 1972). Thus, the phosphorus residence time may be less than the 6-yr value computed on the basis of the 0.01 mg L^{-1} value for the mean annual concentration of the lake.

CONCLUSION

The phosphorus residence time model is potentially useful for assessing a variety of lake rehabilitation procedures. Effects of improved waste water treatment may be simulated by reducing the influx concentration, c_i; the impact of diversion projects can be estimated by altering both c_i, and Q; while in-lake schemes such as alum treatment, artificial destratification or hypolimnetic aeration may be assessed by modifying k. As attempts at lake renewal increase and more is learned about the phosphorus balance in lakes, the model will become more refined. Nonetheless, the phosphorus residence time model described above, though admittedly crude, does provide a simple, easily used and generally realistic basis for predicting the result and rate of many lake renewal endeavors.

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